Gaugino Condensation and the Vacuum Expectation Value of the Dilaton[*]

A. Niemeyer[†] and H. P. Nilles[†,‡]

[†] Physik Department
Technische Universität München
D–85748 Garching, Germany

[‡] Max Planck Institut für Physik
Heisenberg Institut
D–80805 Munich, Germany

ABSTRACT

The mechanism of gaugino condensation has emerged as a prime candidate for supersymmetry breakdown in low energy effective supergravity (string) models. One of the open questions in this approach concerns the size of the gauge coupling constant which is dynamically fixed through the vev of the dilaton. We argue that a nontrivial gauge kinetic function $f(S)$ could solve the potential problem of a runaway dilaton. The actual form of $f(S)$ might be constrained by symmetry arguments.

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One of the prime motivations to consider the supersymmetric extension of the standard model is the stability of the weak scale (M_W) of order of a TeV in the presence of larger mass scales like a GUT-scale of $M_X = 10^{16}\ GeV$ or the Planck scale $M_{Pl} \approx 10^{18}\ GeV$. The size of the weak scale is directly related to the breakdown scale of supersymmetry, and a satisfactory mechanism of supersymmetry breakdown should explain the smallness of M_W/M_{Pl} in a natural way. One such mechanism is based on the dynamical formation of gaugino condensates that has attracted much attention since its original proposal for a spontaneous breakdown of supergravity [1][2]. In this talk we shall address some open questions concerning this mechanism in the framework of low energy effective superstring theories. This work has been done in collaboration with Z. Lalak and appeared in ref. [3][4].

Before addressing these detailed questions let us remind you of the basic facts of this mechanism. For simplicity we shall consider here a pure supersymmetric ($N = 1$) Yang-Mills theory, with the vector multiplet (A_μ, λ) containing gauge bosons and gauge fermions in the adjoint representation of the nonabelian gauge group. Such a theory is asymptotically free and we would therefore (in analogy to QCD) expect confinement and gaugino condensation at low energies [5]. We are then faced with the question whether such a nontrivial gaugino condensate $<\lambda\lambda> \neq 0$ leads to a breakdown of supersymmetry. A first look at the SUSY-transformation on the composite fermion $\lambda\sigma^\mu A_\mu$ [6]

$$\{Q, \lambda\sigma^\mu A_\mu\} = \lambda\lambda + \ldots \tag{1}$$

might suggest a positive answer, but a careful inspection of the multiplet structure and gauge invariance leads to the opposite conclusion. The bilinear $\lambda\lambda$ has to be interpreted as the lowest component of the chiral superfield $W^\alpha W_\alpha = (\lambda\lambda, \ldots)$ and therefore a non-vanishing vev of $\lambda\lambda$ does not break SUSY [7]. This suggestion is supported by index-arguments [8] and an effective Lagrangian approach [9]. We are thus lead to the conclusion that in such theories gaugino condensates form, but do not break global (rigid) supersymmetry.

Not all is lost, however, since we are primarily interested in models with local supersymmetry including gravitational interactions. The weak gravitational force should not interfere with the formation of the condensate; we therefore still assume $<\lambda\lambda> = \Lambda^3 \neq 0$. This expectation is confirmed by the explicit consideration of the effective Lagrangian of ref. [1] in the now locally supersymmetric framework. We here consider a composite chiral superfield $U = (u, \psi, F_u)$ with $u = <\lambda\lambda>$. In this toy model [1][2] we obtain the surprising result that not only $<u> = \Lambda^3 \neq 0$ but also $<F_u> \neq 0$, a signal for supersymmetry breakdown. In fact

$$<F_u> = M_S^2 = \frac{\Lambda^3}{M_{Pl}}, \tag{2}$$

consistent with our previous result that in the global limit $M_{Pl} \rightarrow \infty$ (rigid) supersymmetry is restored. For a hidden sector supergravity model we would choose $M_S \approx 10^{11}\ GeV$ [2].

Still more information can be obtained by consulting the general supergravity Lagrangian of elementary fields determined by the Kähler potential $K(\Phi_i, \Phi^{j^*})$, the superpotential $W(\Phi_i)$ and the gauge kinetic function $f(\Phi_i)$ for a set of chiral superfields $\Phi_i = (\phi_i, \psi_i, F_i)$. Non-vanishing vevs of the auxiliary fields F_i would signal a breakdown of supersymmetry. In standard supergravity notation these fields are given by

$$F_i = \exp(G/2)(G^{-1})_i^j G_j + \frac{1}{4}\frac{\partial f}{\partial \Phi_k}(G^{-1})_i^k \lambda\lambda + \ldots, \tag{3}$$

where the gaugino bilinear appears in the second term [10]. This confirms our previous argument that $<\lambda\lambda> \neq 0$ leads to a breakdown of supersymmetry, however, we obtain a new condition: $\partial f/\partial \Phi_i$ has to be nonzero, i.e. the gauge kinetic function $f(\Phi_i)$ has to be nontrivial. In the fundamental action $f(\Phi_i)$ multiplies $W_\alpha W^\alpha$ which in components leads to a form $\mathrm{Re}f(\phi_i)F_{\mu\nu}F^{\mu\nu}$ and tells us that the gauge coupling is field dependent. For simplicity we consider here one modulus field M with

$$<\mathrm{Re}f(M)> = 1/g^2. \tag{4}$$

This dependence of f on the modulus M is very crucial for SUSY breakdown via gaugino condensation. $\partial f/\partial M \neq 0$ leads to $F_M \approx \Lambda^3/M_{Pl}$ consistent with previous considerations. The goldstino is the fermion in the $f(M)$ supermultiplet. In the full description of the theory it might mix with a composite field, but the inclusion of the composite fields should not alter the qualitative behaviour discussed here. An understanding of the mechanism of SUSY breakdown via gaugino condensation is intimately related to the question of a dynamical determination of the gauge coupling constant as the vev of a modulus field. We would hope that in a more complete theory such questions could be clarified in detail.

One candidate of such a theory is the $E_8 \times E_8$ heterotic string. The second E_8 (or a subgroup thereof) could serve as the hidden sector gauge group and it was soon found [11] that there we have nontrivial $f = S$ where S represents the dilaton superfield. The heterotic string thus contains all the necessary ingredients for a successful implementation of the mechanism of SUSY breakdown via gaugino condensation [12][13]. Also the question of the dynamical determination of the gauge coupling constant can be addressed. A simple reduction and truncation from the $d = 10$ theory leads to the following scalar potential [14]

$$V = \frac{1}{16 S_R T_R^3}\left[|W(\Phi) + 2(S_R T_R)^{3/2}(\lambda\lambda)|^2 + \frac{T_R}{3}\left|\frac{\partial W}{\partial \Phi}\right|^2\right], \tag{5}$$

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where $S_R = \mathrm{Re}S$, $T_R = \mathrm{Re}T$ is the modulus corresponding to the overall radius of compactification and $W(\Phi)$ is the superpotential depending on the matter fields Φ. The gaugino bilinear appears via the second term in the auxiliary fields (3). To make contact with the dilaton field, observe that $<\lambda\lambda> = \Lambda^3$ where Λ is the renormalization group invariant scale of the nonabelian gauge theory under consideration. In the one-loop approximation

$$\Lambda = \mu \exp\left(-\frac{1}{bg^2(\mu)}\right), \tag{6}$$

with an arbitrary scale μ and the β-function coefficient b. This then suggests

$$\lambda\lambda \approx e^{-f} = e^{-S} \tag{7}$$

as the leading contribution (for weak coupling) for the functional f-dependence of the gaugino bilinear[1].

In the potential (5) we can then insert (7) and determine the minimum. In our simple model (with $\partial W/\partial T = 0$) we have a positive definite potential with vacuum energy $E_{vac} = 0$. Suppose now for the moment that $<W(\Phi)> \neq 0$[2].
S will now adjust its vev in such a way that $|W(\Phi) + 2(S_R T_R)^{3/2}(\lambda\lambda)| = 0$, thus

$$|W(\Phi) + 2(S_R T_R)^{3/2} \exp(-S)| = 0. \tag{8}$$

This then leads to broken SUSY with $E_{vac} = 0$ and a fixed value of the gauge coupling constant $g^2 \approx <\mathrm{Re}S>^{-1}$. For the vevs of the auxiliary fields we obtain $F_S = 0$ and $F_T \neq 0$ with important consequences for the pattern of the soft SUSY breaking terms in phenomenologically oriented models [17], which we shall not discuss here in detail.

Thus a satisfactory picture seems to emerge. However, we have just discussed a simplified example. In general we would expect also that the superpotential depends on the moduli, $\partial W/\partial T \neq 0$ and, including this dependence, the modified potential would no longer be positive definite and one would have $E_{vac} < 0$.

But even in the simple case we have a further vacuum degeneracy. For any value of $W(\Phi)$ we obtain a minimum with $E_{vac} = 0$, including $W(\Phi) = 0$. In the latter case this would correspond to $<\lambda\lambda> = 0$ and $S \to \infty$. This is the potential problem of the runaway dilaton. The simple model above does not exclude such a possibility. In fact this problem of the runaway dilaton does not seem just to be a problem of the toy model, but more general. One attempt to avoid this problem was the consideration of

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[1] Relation (7) is of course not exact. For different implementations see [13], [15], [16]. The qualitative behaviour of the potential remains unchanged.

[2] In many places in the literature it is quoted incorrectly that $<W(\Phi)>$ is quantized in units of the Planck length since W comes from H, the field strength of the antisymmetric tensor field B and $H = \mathrm{d}B - \omega_{3Y} + \omega_{3L}$ (ω being the Chern-Simons form) [16]. Quantization is expected for $<\mathrm{d}B>$ but not for H.

several gaugino condensates [18], but it still seems very difficult to produce satisfactory potentials that lead to a dynamical determination of the dilaton for reasonable values of $<S>$. In some cases it even seems impossible to fine tune the cosmological constant to zero. In absence of a completely satisfactory model it is then also difficult to investigate the detailed phenomenological properties of the approach. Here it would be of interest to know the actual size of the vevs of the auxiliary fields $<F_S>$, $<F_T>$ and $<F_U>$. In the models discussed so far one usually finds $<F_T>$ to be the dominant term, but it still remains a question whether this is true in general.

In any case it seems that we need some new ingredient before we can understand the mechanism completely. It is our belief, that the resolution of all these problems comes with a better understanding of the form of the gauge kinetic function f [3][4]. In all the previous considerations one assumed $f = S$. How general is this relation? Certainly we know that in one loop perturbation theory S mixes with T [19], but this is not relevant for our discussion and, for simplicity, we shall ignore that for the moment. The formal relation between f and the condensate is given through $\Lambda^3 \approx e^{-f}$ and we have $f = S$ in the weak coupling limit of string theory. In fact this argument tells us only that

$$\lim_{S \to \infty} f(S) = S. \tag{9}$$

Nonperturbative effects could lead to the situation that f is a very complicated function of S. In fact a satisfactory incorporation of gaugino condensates in the framework of string theory might very well lead to such a complication. In our work [3] we suggested that a nontrivial f-function is the key ingredient to better understand the mechanism of gaugino condensation. We still assume (9) to make contact with perturbation theory. How do we then control e^{-f} as a function of S? In absence of a determination of $f(S)$ by a direct calculation one might use symmetry arguments to make some progress. Let us here consider the presence of a symmetry called S-duality which in its simplest form is given by a $SL(2, Z)$ generated by the transformations

$$S \to S + i, \quad S \to -1/S. \tag{10}$$

Such a symmetry might be realized in two basically distinct ways: the gauge sector could close under the transformation (type I) or being mapped to an additional 'magnetic sector' with inverted coupling constant (type II). In the second case one would speak of strong-weak coupling duality, just as in the case of electric-magnetic duality [20]. Within the class of theories of type I, however, we could have the situation that the f-function is itself invariant under S-duality; i.e. $S \to -1/S$ does not invert the coupling constant since the gauge coupling constant is not given by $\mathrm{Re}S$ but $1/g^2 \approx \mathrm{Re}f$. In view of (9) we would call such a symmetry weak-weak coupling duality. The behaviour of the gauge coupling constant as a function of S is shown in Fig. 1. Our assumption (9) implies that $g^2 \to 0$ as $\mathrm{Re}S \to \infty$ and by S-duality g^2 also vanishes

for $S \to 0$, with a maximum somewhere in the vicinity of the self-dual point $S = 1$. Observe that $S \approx 1$ in this situation does not necessarily imply strong coupling, because $g^2 \approx 1/\text{Re} f$ and even for $S \approx 1$, $\text{Re} f$ could be large and $g^2 << 1$, with perturbation theory valid in the whole range of S. Of course, nonperturbative effects are responsible for the actual form of $f(S)$.
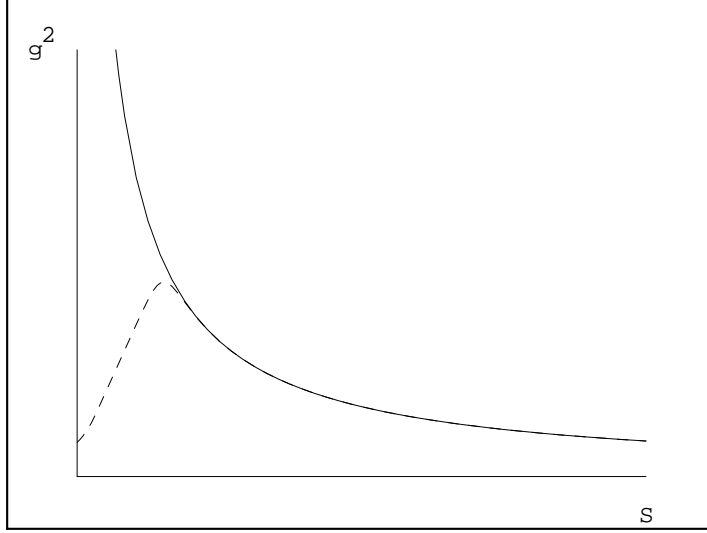

Fig. 1 - Coupling constant g^2 as the function of S in type-I models (dashed) vs g^2 given by $f = S$

To examine the behaviour of the scalar potential in this approach, let us consider a simple toy model, with chiral superfield $U = Y^3 = (\lambda\lambda, \ldots)$ as well as S and T. We have to choose a specific example of a gauge kinetic function which is invariant under the S-duality transformations. Different choices are possible, the simplest is given by

$$f = \frac{1}{2\pi} \ln(j(S) - 744), \tag{11}$$

$j(S)$ being the usual generator of modular invariant functions. This function behaves like S in the large S-limit. If we assume a type I-model where the gauge sector is closed under S-duality, then we also have to assume that the gaugino condensate does not transform under S-duality (because of the $fW^\alpha W_\alpha$-term in the Lagrangian)[3]. Under these conditions an obvious candidate for the superpotential is just the standard Veneziano-Yankielowicz superpotential (extended to take into account the usual T-duality, which we assume to be completely independent from S-duality) [9][21]

$$W = Y^3(f + 3b \ln \frac{Y\eta^2(T)}{\mu} + c). \tag{12}$$

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[3]For type I-models it was shown in [3] that one can always redefine the gauge kinetic function and condensate in such a way that this holds.

This is clearly invariant under S-duality. Therefore we then cannot take the conventional form for the Kähler potential which would be given by

$$K = -\ln(S + \bar{S}) - 3\ln(T + \bar{T} - Y\bar{Y}), \tag{13}$$

since it is not S-dual. To make it S-dual one could introduce an additional $\ln|\eta(S)|^4$ term, giving e.g.

$$K = \ln(S + \bar{S}) - 3\ln(T + \bar{T} - Y\bar{Y}) - \ln|\eta(S)|^4. \tag{14}$$

Because the only relevant quantity is

$$G = K + \ln|W|^2, \tag{15}$$

we can as well put this new term (which is forced upon us because of our demand for symmetry) into the superpotential and take the canonical Kähler function instead, which gives

$$K = \ln(S + \bar{S}) - 3\ln(T + \bar{T} - Y\bar{Y}), \tag{16}$$

$$W = \frac{Y^3}{\eta^2(S)}(f + 3b\ln\frac{Y\eta^2(T)}{\mu} + c), \tag{17}$$

where the remarkable similarity to the effective potential for T-dual gaugino condensation $W = W_{inv}/\eta^6(T)$ can be seen more clearly.

This model exhibits a well defined minimum at $<S> = 1$, $<T> = 1.23$ and $<Y> \approx \mu$. Supersymmetry is broken with the dominant contribution being $<F_T> \approx \mu^3$. The cosmological constant is negative.

In contrast to earlier attempts [22] this model fixes the problem of the runaway dilaton and breaks supersymmetry with only a single gaugino condensate. Previous models needed multiple gaugino condensates and (to get realistic vevs for the dilaton) matter fields in complicated representations. We feel that the concept of a nontrivial gauge kinetic function derived (or constrained) by a symmetry is a much more natural way to fix the dilaton and break supersymmetry, especially so because corrections to $f = S$ are expected in any case. Earlier models which included S-duality in different ways (both with and without gaugino condensates) [23][24] were able to fix the vev of the dilaton but did not succeed in breaking supersymmetry. An alternative mechanism to fix the vev of the dilaton has been discussed in [25].

Of course there are still some open questions not solved by this approach. The first is the problem of having a vanishing cosmological constant. Whereas early models of gaugino condensation often introduced *ad hoc* terms to guarantee a vanishing vacuum energy, it has been seen to be notoriously difficult to get this out of models based on string inspired supergravity. The only way out of this problem so far has been to introduce a constant term into the superpotential, parameterizing unknown effects.

This approach does not even work in any arbitrary model, but at least in our model the cosmological constant can be made to vanish by adjusting such a constant.

Another question not addressed in this toy model is the mixing of S and T fields which happens at the one-loop level. It is still unknown whether one can keep two independent dualities in this case. In a consistent interpretation our toy model should describe an all-loop effective action. If it is considered to be a theory at the tree-level then the theory is not anomaly free. Introducing terms to cancel the anomaly which arises because of demanding S-duality will then destroy S-duality. At tree-level the theory therefore cannot be made anomaly free.

An additional interesting question concerns the vevs of the auxiliary fields, i.e. which field is responsible for supersymmetry breakdown. In all models considered so far (multiple gaugino condensates, matter, S-duality) it has always been F_T which dominates all the other auxiliary fields. It has not been shown yet that this is indeed a generic feature. The question is an important one, since the hierarchy of the vevs of the auxiliary fields is mirrored in the structure of the soft SUSY breaking terms of the MSSM [17]. We want to argue that there is at least no evidence for F_T being generically large in comparison to F_S, because all of the models constructed so far (including our toy model) are designed in such a way that $<F_S> = 0$ by construction at the minimum (at least at tree-level for the other models). In fact, if one extends our model with a constant in the superpotential (see above), then $<F_S>$ increases with the constant (but does not become as large as $<F_T>$).

Of course there are still some assumptions we made by considering this toy model. We assumed that there is weak coupling in the large S limit which is an assumption because the nonperturbative effects are unknown (at tree-level it can be calculated that $f = S$). In addition it is clear that the standard form we take for the Kähler potential does not include nonperturbative effects and thus could be valid only in the weak coupling approximation (this is of course related to our choice of the superpotential). Of course an equally valid assumption would be that nonperturbative effects destroy the calculable tree-level behaviour even in the weak coupling region. The model of ref. [23] could be re-interpreted in that sense (they do not consider gaugino condensates and the gauge kinetic function, but their S-dual scalar potential goes to infinity for $S \to \infty$). We choose not to make this assumption, because it is equivalent to the statement that the whole perturbative framework developed so far in string theory is wrong. Again it should be emphasized here that the S-duality considered is not a strong-weak coupling duality but a weak-weak coupling duality. In type II-models one has a duality between strong and weak coupling [3].

An additional problem could be the size of the gauge coupling constant. If $f = S$ and $<S> = 1$ then the large value of the gauge coupling constant does not fit the low scale of gaugino condensation necessary for phenomenologically realistic supersymmetry breaking ($10^{13} \, GeV$). However if $f = S$ only in the weak coupling limit then one can have $<f> >> 1$ and thus $g^2 << 1$ even in the region $S = O(1)$. Therefore in our

model $<S> = 1$ is consistent with the demand for a small gauge coupling constant, whereas in models with $f = S$ a much larger (and therefore more unnatural) $<S>$ is needed.

To summarize we find that the choice of a nontrivial f-function (motivated by a symmetry requirement) gives rise to a theory where supersymmetry breaking is achieved by employing only a single gaugino condensate. The cosmological constant turns out to be negative, but can be adjusted by a simple additional constant in the superpotential. The vevs of all fields are at natural orders of magnitude and due to the nontrivial gauge kinetic function the gauge coupling constant can be made small enough to give a realistic picture.

Acknowledgments

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